OPTIMAL STATISTICAL DECISIONS IN A NEW PRODUCT LIFETIME TESTING

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ABSTRACT

This paper considers the problem that can be stated as follows. A new product is submitted for lifetime testing. The product will be accepted if a random sample of n items shows r or fewer failures in performance testing. We want to know whether to stop the test before it is completed if the results of the early observations are unfavorable. A suitable stopping decision saves the cost of the waiting time for completion. On the other hand, an incorrect stopping decision causes an unnecessary design change and a complete rerun of the test. It is assumed that the redesign would improve the product to such an extent that it would definitely be accepted in a new lifetime testing. The paper presents a stopping rule based on the statistical estimation of total costs involved in the decision to continue beyond an early failure. Sampling is both expensive and time consuming. Hence, there are situations where it is more efficient to take samples sequentially, as opposed to all at one time, and to define a stopping rule to terminate the sampling process. One of these situations is considered in the paper. The practical applications of the stopping rules are illustrated with examples.

INTRODUCTION

The theory of stopping rules has its roots in the study of the optimality properties of the sequential probability ratio test of Wald and Wolfowitz (1948) and Arrow, Blackwell and Girshick (1949). The essential idea in both of these papers was to create a formal Bayes problem.

The formal Bayes problem is what we would now call an optimal stopping problem. A decision maker observes an adapted sequence $\{R_n, \mathcal{F}_n, n \geq \mathrm{I}\}$, with $E\{|R_n|\} < \infty$ for all n, where $\mathcal{F}_n$ denotes the σ-algebra generated by a sequence of rewards $R_1, \ldots, R_n$. At each time n a choice is to be made, to stop sampling and collect the currently available reward, R_n, or continue sampling in the *expectation* of collecting a larger reward in the future. An optimal stopping rule N is one that maximizes the expected reward, $E\{R_n\}$. The key to finding an optimal or close to optimal stopping rule is the family of equations

$$Z_n = \max\,(R_n, E\{Z_{n+1}; \mathcal{F}_n\}), \quad n=1, 2, \ldots . \tag{1}$$

The informal interpretation of Z_n, is that it is the most one can expect to win if one has already reached stage n; and equations (1) say that this quantity is the maximum of what one can win by stopping at the nth stage and what one can expect to win by taking at least one more observation and proceeding optimally thereafter. The plausible candidate for an optimal rule is to stop with

$$N = \min\{\, n : R_n \geq E\{Z_{n+1}; \mathcal{F}_n\}\}, \tag{2}$$

that is, stop as soon as the current reward is at least as large as the most that one can expect to win by continuing. Equations (1) show that $\{Z_n, \mathcal{F}_n\}$ is a supermartingale, while $\{Z_{\min(N,n)}, \mathcal{F}_n\}$, is a martingale. The equations do not have a unique solution, but in the case where the index n is bounded, say $1 \leq n \leq m$ for some given value of m, the solution of interest satisfies $Z_m = R_m$. Hence (1) can be solved and the optimal stopping rule can be found by "backward induction." The general strategy of optimal stopping theory is to approximate the case where no bound m exists by first imposing such a bound, solving the bounded problem and then letting $m \to \infty$. For reviews of the many variations on this problem and the extensive related literature, see Freeman (1983), Petrucelli (1988) and Samuels (1991).

For illustration of the stopping problem, consider the Bayesian sequential estimation problem of a binomial parameter under quadratic loss and constant observation cost. Suppose that the unknown binomial parameter p is assigned a beta prior distribution with integer parameters (a,b) so that

$$\pi(p; a, b) = \frac{(b-1)!}{(a-1)!(b-a-1)!}\, p^{a-1}(1-p)^{b-a-1}, \quad 0 < p < 1. \tag{3}$$

The posterior distribution of p having observed s successes in n trials is simply $\pi(p;s+a,n+b)$ (Raiffa and Schlaifer 1968); hence the result of sampling may be represented as a plot of $s+a$ against $n+b$ which stops when the stopping boundary is reached. If $a=1$, $b=2$, the uniform prior, is taken as the origin, sample paths for any other proper prior will start at the point $(a-1,b-2)$. Consequently stopping boundaries will be obtained using the uniform prior.

Suppose that the loss in estimating p by d is $\vartheta(p-d)^2$ where ϑ is a constant giving loss in terms of cost. Then the Bayes estimator is the current prior mean $(s+1)/(n+2)$ and the Bayes risk is

$$B(s,n)=\frac{\vartheta(s+1)(n-s+1)}{(n+2)^2(n+3)}. \quad (4)$$

At a point (s,n) let $D(s,n)$ be the risk of taking one further observation at a cost c and $M(s,n)$ be the minimum risk, then the dynamic programming equations giving the partition of the (s,n) plane into stopping and continuation points are

$$M(s,n)=\min\{B(s,n),D(s,n)\}, \quad (5)$$

where

$$D(s,n)=c+\frac{s+1}{n+2}M(s+1,n+1)+\frac{n-s+1}{n+2}M(s,n+1). \quad (6)$$

The equations are similar to those of Lindley and Barnett (1965) and Freeman (1970, 1972, 1973).

The optimal decision at each point is obtained by working back from a maximum sample size, which is approximately $[(1/2)\sqrt{\vartheta/c}]-2$.

A suboptimal stopping point (s,n) is defined as a first stopping point for fixed s if $(s,n-1)$ is a continuation point, in this case

$$D(s,n-1)=c+\frac{s+1}{n+1}M(s+1,n)+\frac{n-s}{n+1}B(s,n)$$

$$\le c+\frac{s+1}{n+1}B(s+1,n)+\frac{n-s}{n+1}B(s,n)=D^{\bullet}(s,n-1). \quad (7)$$

A lower bound for the sample size n above may now be found from (7) by setting $B(s,n-1)\ge D^{\bullet}(s,n-1)$. This leads to

$$[(n+2)(n+1)]^2\le(\vartheta/c)(s+1)(n-s). \quad (8)$$

The optimal stopping boundary starts at $s=0$ and n, and from (8) it may be shown that this sample size is at least $[(\vartheta/c)^{1/3}]-3$.

The approximate design obtained by (8) will be termed a one step ahead design. Both designs will obviously stop at the same maximum number of observations N, and will give the same decision after $(N-1)$ observations. The one step ahead design gives stopping boundaries, which will lie inside those of the optimal. The one step ahead design is similar to the modified Bayes rule of Amster (1963) and has been used by El-Sayyad and Freeman (1973) to estimate a Poisson process rate.

The present research investigates the frequentist (non-Bayesian) stopping rules. In this paper, stopping rules in fixed-sample testing as well as in sequential-sample testing are discussed.

ASSUMPTIONS AND COST FUNCTIONS IN FIXED-SAMPLE TESTING

Let c_1 be the cost per hour of conducting the test, c_2 be the total cost of redesign (including the time required to implement it). The cost of redesign c_2 is undoubtedly the most difficult to estimate. This cost is to include whatever redesigns are necessary to make the probability of failure on rerun negligible. To simplify the mathematics, it is assumed that unnecessary design changes, caused by incorrectly abandoning the test, will also have a beneficial effect on performance. This assumption appears warranted for many electronic and mechanical systems, where the introduction of redundancies, higher-quality components, etc., can always be expected to improve reliability.

It will be assumed in this section that the times of interest to the decision maker are restricted to those where a failure has just occurred.

Let $X_1, X_2, \ldots, X_k$ be the ordered sample of observations from a population with lifetime distribution $f(x;\theta)$. Let $\widehat{\theta}$ be the maximum-likelihood estimate of θ based upon the first k order statistics $(X_1, \ldots, X_k)\equiv X^k$. Let $g(x_1, x_2, \ldots, x_k;\theta)$ be the joint density of the k observations, $g(x_1, x_2, \ldots, x_k, x_{r+1};\theta)$ be the joint density of the first k and $(r+1)st$ order statistics, and $f(x_{r+1};x^k,\theta)$ be the conditional density of the r*th* order statistic.

If τ_0 is the life specified as acceptable, then the probability of passing the test after x_k has been observed may be estimated as

$$\widehat{p}_{pas}=\int_{\tau_0}^{\infty}f(x_{r+1};x^k,\widehat{\theta})dx_{r+1}, \quad (9)$$

where

$$f(x_{r+1};x^k,\widehat{\theta})=\frac{g(x_1,\ldots,x_k,x_{r+1};\widehat{\theta})}{g(x_1,\ldots,x_k;\widehat{\theta})}. \quad (10)$$

The cost of abandoning the test is

$$c_{abandoning} = c_1\tau_0 + c_2\,. \tag{11}$$

The estimated cost of continuation of the test is given by

$$\hat{c}_{continuing} = \int_{x_k}^{\tau_0} [c_1(x_{r+1} - x_k) + c_1\tau_0 + c_2] f(x_{r+1}; x^k, \hat{\theta}) dx_{r+1}$$

$$+ \int_{\tau_0}^{\infty} c_1(\tau_0 - x_k) f(x_{r+1}; x^k, \hat{\theta}) dx_{r+1}$$

$$= c_1 \int_{x_k}^{\tau_0} x_{r+1} f(x_{r+1}; x^k, \hat{\theta}) dx_{r+1}$$

$$+(1 - \hat{p}_{pas})[c_1(\tau_0 - x_k) + c_2] + \hat{p}_{pas}[c_1(\tau_0 - x_k)]$$

$$= c_1 \int_{x_k}^{\tau_0} x_{r+1} f(x_{r+1}; x^k, \hat{\theta}) dx_{r+1} + c_1\tau_0 + c_2 - c_1 x_k - \hat{p}_{pas} c_2. \tag{12}$$

STOPPING RULE IN FIXED-SAMPLE TESTING

The decision rule will be based on the relative magnitude of $c_{abandoning}$ and $\hat{c}_{continuing}$. The simplest rule would be:

If $\hat{c}_{continuing} < c_{abandoning}$, i.e., if

$$\int_{x_k}^{\tau_0} x_{r+1} f(x_{r+1}; x^k, \hat{\theta}) dx_{r+1} < x_k + \hat{p}_{pas} \frac{c_2}{c_1}, \tag{13}$$

continue the present test;

If $\hat{c}_{continuing} \ge c_{abandoning}$, i.e., if

$$\int_{x_k}^{\tau_0} x_{r+1} f(x_{r+1}; x^k, \hat{\theta}) dx_{r+1} \ge x_k + \hat{p}_{pas} \frac{c_2}{c_1}, \tag{14}$$

abandon the present test and initiate a redesign.

ESTIMATING THE PROBABILITY OF PASSING THE FIXED-SAMPLE TEST

Evaluation of the cost functions for the lifetime-testing model requires, even for relatively simple probability distributions, the evaluation of some complicated integrals that cannot always be obtained in closed form. For example, using the one-parameter exponential model for lifetime distribution, we have

$$f(x;\theta) = \frac{1}{\theta}\exp\left(-\frac{x}{\theta}\right), \quad x \ge 0, \tag{15}$$

$$F(x;\theta) = 1 - \exp\left(-\frac{x}{\theta}\right). \tag{16}$$

Therefore,

$$g(x_1, ..., x_k; \theta) = \frac{n!}{(n-k)!}\frac{1}{\theta^k} \times \exp\left(-\sum_{i=1}^{k}\frac{x_i}{\theta}\right)\left[\exp\left(-\frac{x_k}{\theta}\right)\right]^{n-k}; \tag{17}$$

$$g(x_1, ..., x_k, x_{r+1}; \theta) = \frac{n!}{(r-k)!(n-r-1)!}\frac{1}{\theta^{k+1}} \times \left[\exp\left(-\frac{x_k}{\theta}\right) - \exp\left(-\frac{x_{r+1}}{\theta}\right)\right]^{r-k} \times \exp\left(-\sum_{i=1}^{k}\frac{x_i}{\theta}\right)\left[\exp\left(-\frac{x_{r+1}}{\theta}\right)\right]^{n-r}. \tag{18}$$

The maximum likelihood estimate for θ is

$$\hat{\theta} = \frac{\sum_{i=1}^{k} x_i + (n-k)x_k}{k}. \tag{19}$$

Replacing θ by $\hat{\theta}$ in the density functions and simplifying, we obtain

$$\hat{p}_{pas} = \int_{\tau_0}^{\infty} \frac{(n-k)!}{(r-k)!(n-r-1)!} \times \frac{\left[\exp\left(-\frac{x_k}{\hat{\theta}}\right) - \exp\left(-\frac{x_{r+1}}{\hat{\theta}}\right)\right]^{r-k}}{\left[\exp\left(-\frac{x_k}{\hat{\theta}}\right)\right]^{n-k}} \times \frac{1}{\hat{\theta}}\left[\exp\left(-\frac{x_{r+1}}{\hat{\theta}}\right)\right]^{n-r} dx_{r+1}. \tag{20}$$

If we write

$$\left[\exp\left(-\frac{x_k}{\hat{\theta}}\right)\right]^{n-k} = \left[\exp\left(-\frac{x_k}{\hat{\theta}}\right)\right]^{r-k}\left[\exp\left(-\frac{x_k}{\hat{\theta}}\right)\right]^{n-r}, \tag{21}$$

then it is clear that

$$\widehat{p}_{pas} = \int_{\tau_0}^{\infty} \frac{(n-k)!}{(r-k)!(n-r-1)!}$$

$$\times \frac{1}{\widehat{\theta}} \left[1 - \frac{\exp\left(-\frac{x_{r+1}}{\widehat{\theta}}\right)}{\exp\left(-\frac{x_k}{\widehat{\theta}}\right)} \right]^{r-k} \left[\frac{\exp\left(-\frac{x_{r+1}}{\widehat{\theta}}\right)}{\exp\left(-\frac{x_k}{\widehat{\theta}}\right)} \right]^{n-r} dx_{r+1}. \quad (22)$$

The change of variable

$$v = \frac{\exp\left(-\frac{x_{r+1}}{\widehat{\theta}}\right)}{\exp\left(-\frac{x_k}{\widehat{\theta}}\right)} \quad (23)$$

leads to

$$\widehat{p}_{pas} = \int_0^{\exp\left(-\frac{\tau_0 - x_k}{\widehat{\theta}}\right)} \frac{(n-k)!}{(r-k)!(n-r-1)!} v^{n-r-1}(1-v)^{r-k}\, dv. \quad (24)$$

Thus, $\widehat{p}_{pas}$ is equivalent to the cumulative beta distribution with parameters (n-r, r-k+1).

The situation for the Weibull distribution,

$$f(x;\sigma,\delta) = \frac{\delta}{\sigma} x^{\delta-1} \exp\left(-\frac{x^\delta}{\sigma}\right), \quad x \ge 0;$$

$$F(x;\sigma,\delta) = 1 - \exp\left(-\frac{x^\delta}{\sigma}\right), \quad (25)$$

is much the same, except that we make the change of variable

$$v = \frac{\exp\left(-\frac{x_{r+1}^{\widehat{\delta}}}{\widehat{\sigma}}\right)}{\exp\left(-\frac{x_k^{\widehat{\delta}}}{\widehat{\sigma}}\right)}. \quad (26)$$

The maximum likelihood estimates $\widehat{\sigma}$ and $\widehat{\delta}$ of the parameters σ and δ, respectively, required in (26), can only be obtained by iterative methods.

The appropriate likelihood equations for $X_1, \ldots, X_k$ are

$$\frac{\partial L}{\partial \sigma} = 0 = -\frac{k}{\sigma} + \frac{1}{\sigma^2}\left[\sum_{i=1}^{k} x_i^\delta + (n-k)x_k^\delta\right], \quad (27)$$

$$\frac{\partial L}{\partial \delta} = 0 = \frac{k}{\delta} + \sum_{i=1}^{k} x_i - \frac{1}{\sigma}\left[\sum_{i=1}^{k} x_i^\delta \ln x_i + (n-k)x_k^\delta \ln x_k\right]. \quad (28)$$

Now $\widehat{\sigma}$ and $\widehat{\delta}$ can be found from solution of

$$\widehat{\sigma} = \frac{\sum_{i=1}^{k} x_i^{\widehat{\delta}} + (n-k)x_k^{\widehat{\delta}}}{k} \quad (29)$$

and

$$\widehat{\delta} = \left[\begin{array}{l} \left(\sum_{i=1}^{k} x_i^{\widehat{\delta}} \ln x_i + (n-k)x_k^{\widehat{\delta}} \ln x_k\right) \\ \times \left(\sum_{i=1}^{k} x_i^{\widehat{\delta}} + (n-k)x_k^{\widehat{\delta}}\right)^{-1} - \frac{1}{k}\sum_{i=1}^{k} \ln x_i \end{array} \right]^{-1}. \quad (30)$$

The method described above is quite general and works well for a11 closed-form or tabulated cumulative distribution functions, so that numerical integration techniques are not needed for calculating $\widehat{p}_{pas}$. It is easy to see that the general case would involve a change of variable

$$v = \frac{1 - F(x_{r+1})}{1 - F(x_k)}, \quad (31)$$

where, of course, x_k is a constant.

EXAMPLES

Example 1

An electronic component is required to pass a performance test of 500 hours. The specification is that 20 randomly selected items shall be placed on test simultaneously, and 5 failures or less shall occur during 500 hours. The cost of performing the test is \$105 per hour. The cost of redesign is \$5000. Assume that the failure distribution follows a one-parameter exponential model. Three failures are observed at 80, 220, and 310 hours. Should the test be continued?
We have

$$\widehat{\theta} = \frac{80 + 220 + 310 + 17 \times 310}{3} = 1960 \text{ hours}; \quad (32)$$

$$\widehat{p}_{pas} = \int_{500}^{\infty} \frac{17!}{2!14!} \frac{\left[\exp\left(-\frac{310}{1960}\right) - \exp\left(-\frac{x_6}{1960}\right)\right]^2}{\left[\exp\left(-\frac{310}{1960}\right)\right]^{17}}$$

$$\times \frac{1}{1960}\left[\exp\left(-\frac{x_6}{1960}\right)\right]^{15} dx_6 = 0.79665; \qquad (33)$$

Since

$$\int_{x_k}^{\tau_0} x_{r+1} f(x_{r+1}; x^k, \hat{\theta}) dx_{r+1} = 430.05 \text{ hours}$$

$$> x_k + \hat{p}_{pas}\frac{c_2}{c_1} = 310 + 0.79665\frac{5000}{105} = 347.94 \text{ hours}, \qquad (34)$$

abandon the present test and initiate a redesign.

Example 2

Consider the following problem. **A** specification for an automotive hood latch is that, of 30 items placed on test simultaneously, ten or fewer shall fall during 3000 cycles of operation. The cost of performing the test is $2.50 per cycle. The cost of redesign is $8500. Seven failures are observed at 48, 300, 315, 492, 913, 1108, and 1480 cycles. Shall the test be continued beyond the 1480th cycle? Assume a Weibull density function for failures.

We have $\hat{\sigma} = 2766.6$ and $\hat{\delta} = 0.9043$. In turn, these estimates yield $\hat{p}_{pas}$=0.25098.

Since

$$\int_{x_k}^{\tau_0} x_{r+1} f(x_{r+1}; x^k, \hat{\theta}) dx_{r+1} = 1877.6 \text{ hours}$$

$$< x_k + \hat{p}_{pas}\frac{c_2}{c_1} = 1480 + 0.25098\frac{8500}{2.5} = 2333.33 \text{ hours}, \qquad (35)$$

continue the present test.

STOPPING RULE IN SEQUENTIAL-SAMPLE TESTING

At the planning stage of a statistical investigation the question of sample size (n) is critical. For such an important issue, there is a surprisingly small amount of published literature. Engineers who conduct reliability tests need to choose the sample size when designing a test plan. The model parameters and quantiles are the typical quantities of interest. The large-sample procedure relies on the property that the distribution of the t-like quantities is close to the standard normal in large samples. To estimate these quantities the maximum likelihood method is often used. The large-sample procedure to obtain the sample size relies on the property that the distribution of the above quantities is close to standard normal in large samples. The normal approximation is only first order accurate in general. When sample size is not large enough or when there is censoring, the normal approximation is not an accurate way to obtain the confidence intervals. Thus sample size determined by such procedure is dubious.

Sampling is both expensive and time consuming. Hence, there are situations where it is more efficient to take samples sequentially, as opposed to all at one time, and to define a stopping rule to terminate the sampling process. The case where the entire sample is drawn at one instance is known as "fixed sampling". The case where samples are taken in successive stages, according to the results obtained from the previous samplings, is known as "sequential sampling".

Taking samples sequentially and assessing their results at each stage allows the possibility of stopping the process and reaching an early decision. If the situation is clearly favorable or unfavorable (for example, if the sample shows that a widget's quality is definitely good or poor), then terminating the process early saves time and resources. Only in the case where the data is ambiguous do we continue sampling. Only then do we require additional information to take a better decision.

In this section, the following optimal stopping rule for determining the efficient sample size sequentially under assigning warranty period is proposed.

Stopping Rule on the Basis of the Expected Beneficial Effect

Suppose the random variables $X_1, X_2, \ldots,$ all from the same population, are observed sequentially and follow the two-parameter Weibull fatigue-crack initiation lifetime distribution

$$f(x;\sigma,\delta) = \frac{\delta}{\sigma}\left(\frac{x}{\sigma}\right)^{\delta-1}\exp\left[-\left(\frac{x}{\sigma}\right)^{\delta}\right], \quad x \ge 0, \sigma > 0, \delta > 0. \qquad (36)$$

where $f(.)$ is a probability density function of the fatigue-crack initiation lifetime X, σ is a scale parameter, δ is a shape parameter (or slope). After the nth observation ($n \ge n_0$, where n_0 is the initial sample size needful to estimate the unknown parameters of the underlying probability model for the data) the experimenter can stop and receive the beneficial effect on performance,

$$c_1 h^{\text{PL}}_{(1:m);\alpha} - cn, \qquad (37)$$

where c_1 is the unit value of the lower conditional $(1-\alpha)$ prediction limit (warranty period) $h^{\text{PL}}_{(1:m);\alpha} \equiv h^{\text{PL}}_{(1:m);\alpha}(x^n)$, $x^n = (x_1, \ldots, x_n)$, and c is the sampling cost.

Below a rule is given to determine if the experimenter should stop in the nth observation, x_n, or if he should continue until the (n+l)st observation, X_{n+1}, at which time he is faced with this decision all over again.

Consider $h^{\mathrm{PL}}_{(1:m);\alpha}(X_{n+1}, x^n)$ as a function of the random variable X_{n+1}, when x_1, …, x_n are known, then it can be found its expected value

$$E\left\{h^{\mathrm{PL}}_{(1:m);\alpha}(X_{n+1}, x^n); x^n\right\}$$

$$= \int_0^\infty \int_0^\infty h^{\mathrm{PL}}_{(1:m);\alpha}(x_{n+1}, x^n) f(x_{n+1}, v; x^n) dx_{n+1} dv. \qquad (38)$$

where

$$f(x_{n+1}, v; x^n)$$

$$= \frac{n v^{n-2} e^{v\hat{\delta}\sum_{i=1}^{n}\ln(x_i/\hat{\beta})} \, v e^{v\hat{\delta}\ln\left(\frac{x_{n+1}}{\hat{\sigma}}\right)} \hat{\delta}\, x_{n+1}^{-1}}{\int_0^\infty v^{n-2} e^{v\hat{\delta}\sum_{i=1}^{n}\ln\left(\frac{x_i}{\hat{\sigma}}\right)} \left(\sum_{i=1}^{n} e^{v\hat{\delta}\ln\left(\frac{x_i}{\hat{\sigma}}\right)}\right)^{-n} dv}$$

$$\times \left(e^{v\hat{\delta}\ln\left(\frac{x_{n+1}}{\hat{\sigma}}\right)} + \sum_{i=1}^{n} e^{v\hat{\delta}\ln\left(\frac{x_i}{\hat{\sigma}}\right)}\right)^{-(n+1)}, \qquad (39)$$

the maximum likelihood estimates $\hat{\delta}$ and $\hat{\sigma}$ of δ and σ, respectively, are determined from the equations:

$$\hat{\delta} = \left[\left(\sum_{i=1}^{n} x_i^{\hat{\delta}} \ln x_i\right)\left(\sum_{i=1}^{n} x_i^{\hat{\delta}}\right)^{-1} - (1/n)\sum_{i=1}^{n} \ln x_i\right]^{-1},$$

$$\hat{\sigma} = \left[(1/n)\sum_{i=1}^{n} x_i^{\hat{\delta}}\right]^{1/\hat{\delta}}, \qquad (40)$$

$\int_0^\infty f(x_{n+1}, v; x^n) dv$ is the conditional probability density function of X_{n+1} (Nechval *et al*. 2007, 2007a).

Now the optimal stopping rule is to determine the expected beneficial effect on performance for continuing

$$c_1 E\left\{h^{\mathrm{PL}}_{(1:m);\alpha}(X_{n+1}, x^n)\right\} - c(n+1) \qquad (41)$$

and compare this with (37).

If

$$c_1\left(E\left\{h^{\mathrm{PL}}_{(1:m);\alpha}(X_{n+1}; x^n); x^n\right\} - h^{\mathrm{PL}}_{(1:m);\alpha}(x^n)\right) > c, \qquad (42)$$

it is profitable to continue;

If

$$c_1\left(E\left\{h^{\mathrm{PL}}_{(1:m);\alpha}(X_{n+1}; x^n); x^n\right\} - h^{\mathrm{PL}}_{(1:m);\alpha}(x^n)\right) \le c, \qquad (43)$$

the experimenter should stop.

CONCLUSIONS

Stopping rules in sample testing can save substantial time and resources, when the case is clear-cut.

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REFERENCES

Amster, S. J. 1963. “A Modified Bayes Stopping Rule”. *The Annals of Mathematical Statistics* 34, 1404-1413.

Arrow, K.J.; D. Blackwell; and M.A. Girshick. 1949. “Bayes and Minimax Solutions of Sequential Decision Problems”. *Econometrica* 17, 213-244.

El-Sayyad, G. M. and P.R. Freeman. 1973. “Bayesian Sequential Estimation of a Poisson Process Rate”. *Biometrika* 60, 289-296.

Freeman, P. R. 1970. “Optimal Bayesian Sequential Estimation of the Median Effective Dose”. *Biometrika* 57, 79-89.

Freeman, P. R. 1972. “Sequential Estimation of the Size of a Population”. *Biometrika* 59, 9-17.

Freeman, P. R. 1973. “Sequential Recapture”. *Biometrika* 60, 141-153.

Freeman, P. R. 1983. “The Secretary Problem and its Extensions: A Review”. *International Statistical Review* 51, 189-206.

Lindley, D. V. and B.N. Barnett. 1965. “Sequential Sampling: Two Decision Problems with Linear Losses for Binomial and Normal Random Variables”. *Biometrika* 52, 507-532.

Nechval, K.N.; N.A. Nechval; G. Berzins; and M. Purgailis. 2007. “Planning Inspections in Service of Fatigue-Sensitive Aircraft Structure Components for Initial Crack Detection”. *Maintenance and Reliability* 35, 76-80.

Nechval, K.N.; N.A. Nechval; G. Berzins; and M. Purgailis. 2007a. “Planning Inspections in Service of Fatigue-Sensitive Aircraft Structure Components Under Crack Propagation”. *Maintenance and Reliability* 36, 3-8.

Petrucelli, J. D. 1988. “Secretary Problem”. In: *Encyclopedia of Statistical Sciences*, S. Kotz and N. Johnson (Eds.). Wiley, New York, Vol. 8, 326-329.

Raiffa, H. and R. Schlaifer. 1968. *Applied Statistical Decision Theory.* Massachusetts Institute of Technology Press.

Samuels, S. M. 1991. “Secretary Problems”. In *Handbook of Sequential Analysis*, B. K. Ghosh and P. K. Sen (Eds.). Dekker, New York, 35-60.

Wald, A. and J. Wolfowitz. 1948. “Optimum Character of the Sequential Probability Ratio Test”. *The Annals of Mathematical Statistics* 19, 326-339.